演習マクロ経済学　第 14 回　講義資料

2016 年 1 月 27 日

※本ハンドアウトに掲載されている Matlab コードは http://khas.bitbucket.org/からダウンロード可能

1 RBC model

1.1 モデル

$$\max_{c_t, k_{t+1}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \Psi \frac{h_t^{1+\psi}}{1+\psi} \right),$$

$$\text{s.t.} \quad y_t = c_t + k_{t+1} - (1-\delta)k_t,$$

$$y_t = a_t k_t^{\alpha} h_t^{1-\alpha},$$

$$\lim_{t \to \infty} \{\beta^t \lambda_t k_{t+1}\} = 0,$$

$$a_t = (1-\rho)\bar{a} + \rho a_{t-1} + \epsilon_t$$

F.O.Cs：

$$1 = \beta \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} \left(\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right]$$

$$\Psi h_t^{\psi} c_t = (1-\alpha) \frac{y_t}{h_t}$$

1.2 定常状態

$$1 = \beta \left(\alpha \frac{y^*}{k^*} + 1 - \delta \right),$$

$$\Psi (h^*)^{\psi} c^* = (1-\alpha) \frac{y^*}{h^*}$$

$$y^* = \bar{a} (k^*)^{\alpha} (h^*)^{1-\alpha},$$

$$y^* = c^* + i^*,$$

$$k^* = i^* + (1-\delta) k^*$$

定常状態の計算

$$\frac{k^*}{y^*} = \frac{\alpha\beta}{1-\beta(1-\delta)} \;\to\; \frac{i^*}{y^*} = \delta\frac{k^*}{y^*} \;\to\; \frac{c^*}{y^*} = 1 - \frac{i^*}{y^*} \;\to\; h^* = \left(\frac{1-\alpha}{\Psi(c^*/y^*)}\right)^{\frac{1}{1+\psi}},$$
$$\to\; y^* = \bar{a}\left(\frac{k^*}{y^*}\right)^{\frac{\alpha}{1-\alpha}} h^* \;\to\; c^* = \left(\frac{c^*}{y^*}\right) y^* \;\to\; i^* = \left(\frac{i^*}{y^*}\right) y^*$$

1.3 モデル（線形化後）

以下の変数は定常状態からの乖離率を表す.

$$c_t = \mathbb{E}_{t-1}c_t + \eta_t^c, \tag{1}$$

$$y_t = \mathbb{E}_{t-1}y_t + \eta_t^y, \tag{2}$$

$$c_t - \mathbb{E}c_{t+1} + [1-\beta(1-\delta)]\left(\mathbb{E}y_{t+1} - k_{t+1}\right) = 0, \tag{3}$$

$$(1+\psi)h_t + c_t - y_t = 0, \tag{4}$$

$$y_t - (1-\alpha)h_t - a_t = \alpha h_t, \tag{5}$$

$$k_{t+1} - \delta k_t = (1-\alpha)k_t, \tag{6}$$

$$y_t - \frac{c^*}{y^*}c_t - \frac{i^*}{y^*}i_t = 0, \tag{7}$$

$$a_t = \rho a_{t-1} + \epsilon_t \tag{8}$$

行列表記：

$$\begin{aligned} AX_t &= BX_{t-1} + C\epsilon_t + D\eta_t, \\ X_t &= \left[\begin{array}{cccccccc} c_t & y_t & h_t & i_t & k_t & \mathbb{E}_t c_{t+1} & \mathbb{E}_t c_{t+1} & a_t \end{array}\right]^\top \\ \eta_t &= \left[\begin{array}{cc} \eta_t^c & \eta_t^y \end{array}\right]^\top \end{aligned}$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -[1-\beta(1-\delta)] & -1 & [1-\beta(1-\delta)] & 0 \\ 1 & -1 & (1+\psi) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & (1-\alpha) & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -\delta & 1 & 0 & 0 & 0 \\ -c^*/y^* & 1 & 0 & -i^*/y^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1-\delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

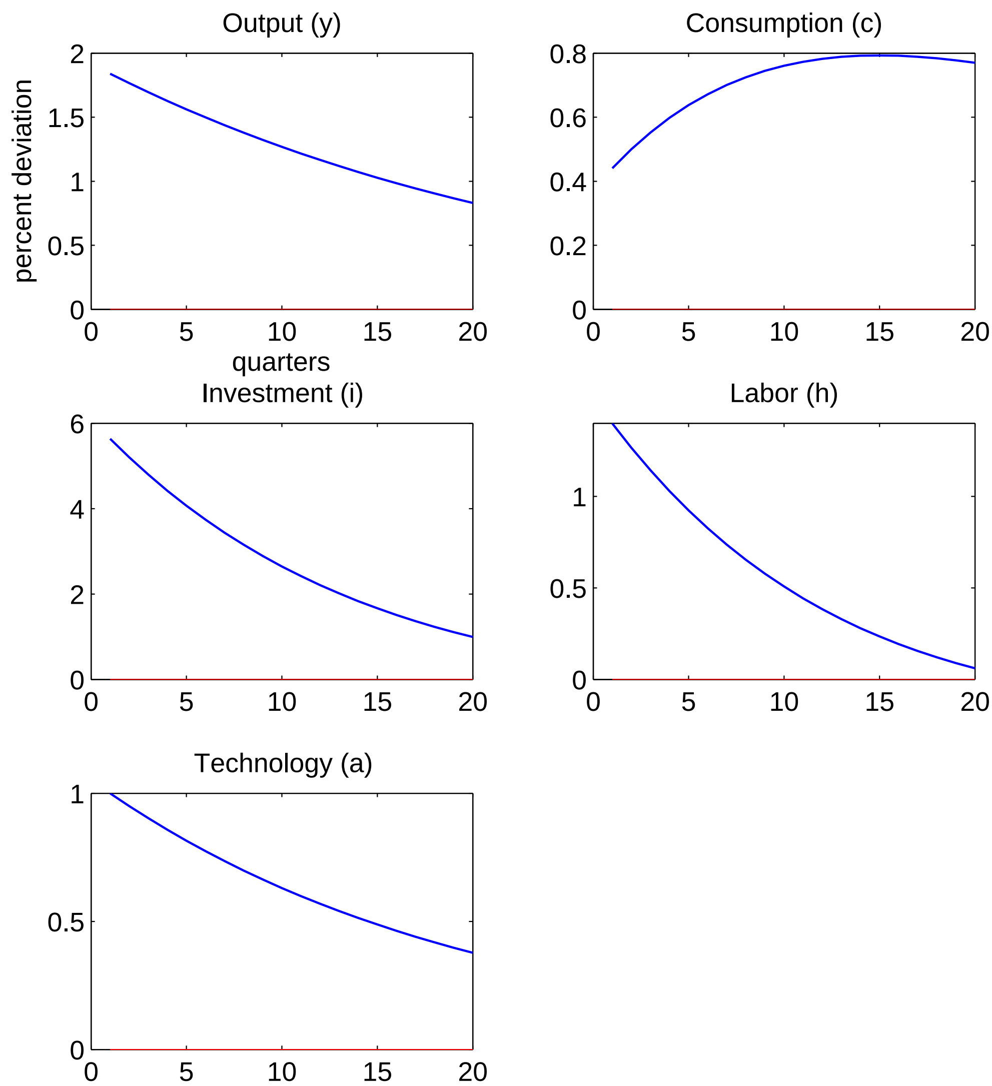

Figure 1: Impulse responses to 1 percent technology shock

2 Optimal Monetary Policy

$$\begin{aligned}
&\min_{\pi_t, x_t, i_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} \left[\pi_t^2 + \lambda_x x_t^2\right], \\
&\text{s.t.} \quad \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t, \\
&\qquad x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n), \\
&\qquad r_t^n = \rho_r r_{t-1}^n + \epsilon_t^r, \\
&\qquad u_t = \rho_u u_{t-1} + \epsilon_t^u
\end{aligned}$$

2.1 Commitment

F.O.Cs:

$$\begin{aligned}
&\pi_t - \phi_{\pi t} = \phi_{\pi t-1}, \\
&x_t + \frac{\kappa}{\lambda_x} \phi_{\pi t} = 0,
\end{aligned}$$

2.2 Discretion

F.O.Cs:

$$\begin{aligned}
&\pi_t - \phi_{\pi t} = 0, \\
&x_t + \frac{\kappa}{\lambda_x} \phi_{\pi t} = 0,
\end{aligned}$$

2.3 System of the model (Commitment)

$$\begin{aligned}
&\pi_t - \beta \mathbb{E}_t \pi_{t+1} - \kappa x_t - u_t = 0, \\
&x_t - \mathbb{E}_t x_{t+1} + \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) = 0, \\
&\pi_t - \phi_{\pi t} = \phi_{\pi t-1}, \\
&x_t + \frac{\kappa}{\lambda_x} \phi_{\pi t} = 0, \\
&r_t^n = \rho_r r_{t-1}^n + \epsilon_t^r, \\
&u_t = \rho_u u_{t-1} + \epsilon_t^u, \\
&\pi_t = \mathbb{E}_{t-1} \pi_t + \eta_t^\pi, \\
&x_t = \mathbb{E}_{t-1} x_t + \eta_t^x
\end{aligned}$$

行列表記：

$$
\begin{aligned}
AX_t &= BX_{t-1} + C\epsilon_t + D\eta_t, \\
X_t &= \left[\begin{array}{cccccccc} \pi_t & x_t & i_t & \phi_{\pi t} & \mathbb{E}_t x_{t+1} & \mathbb{E}_t \pi_{t+1} & r_t^n & u_t \end{array}\right]^\top \\
\eta_t &= \left[\begin{array}{cc} \eta_t^\pi & \eta_t^x \end{array}\right]^\top \\
\epsilon_t &= \left[\begin{array}{cc} \epsilon_t^r & \epsilon_t^u \end{array}\right]^\top
\end{aligned}
$$

$$
A = \begin{bmatrix}
1 & -\kappa & 0 & 0 & 0 & -\beta & 0 & -1 \\
0 & 1 & \sigma & 0 & -1 & -\sigma & -\sigma & 0 \\
1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & \kappa/\lambda_x & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

$$
B = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & \rho_r & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_u \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0
\end{bmatrix}, \quad
C = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
1 & 0 \\
0 & 1 \\
0 & 0 \\
0 & 0
\end{bmatrix}, \quad
D = \begin{bmatrix}
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 0 \\
0 & 1 \\
1 & 0
\end{bmatrix}
$$

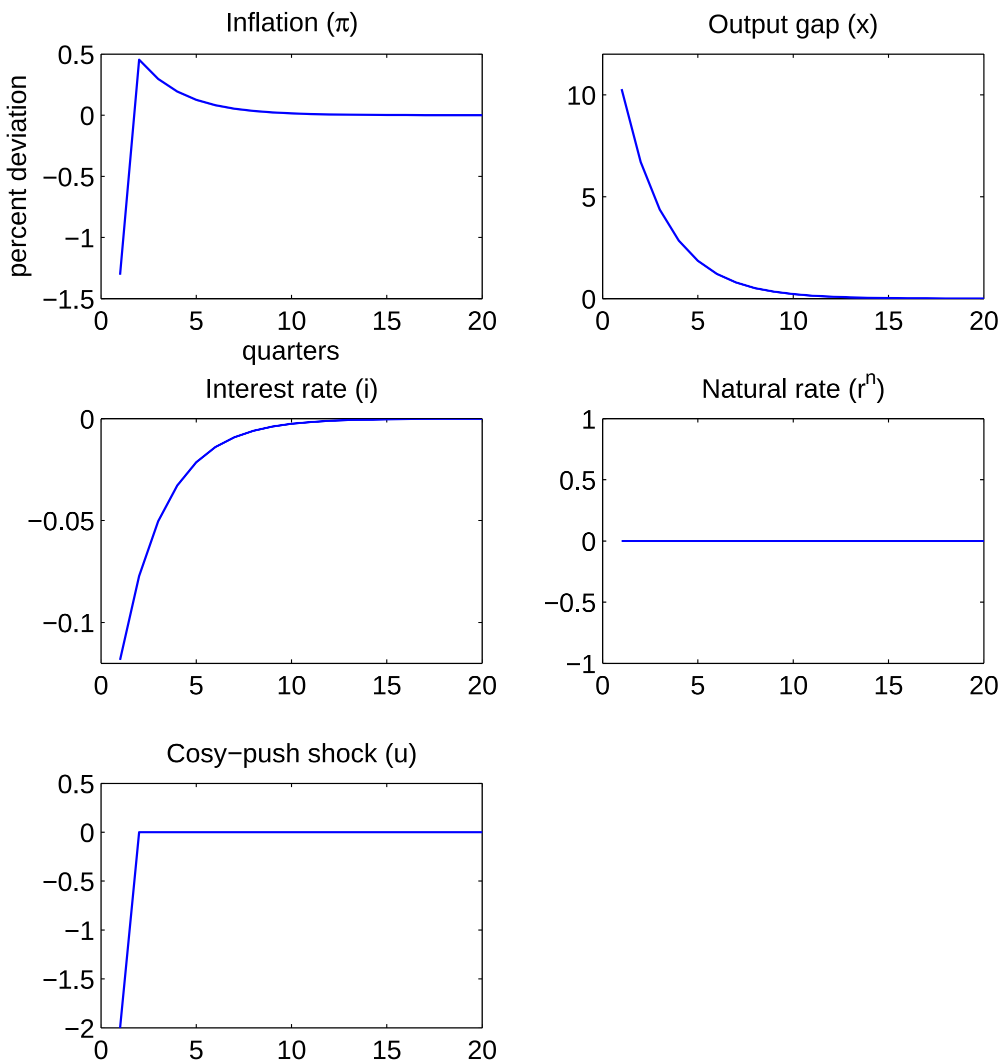

Figure 2: Impulse responses to −2 percent cost-push shock (Commitement)

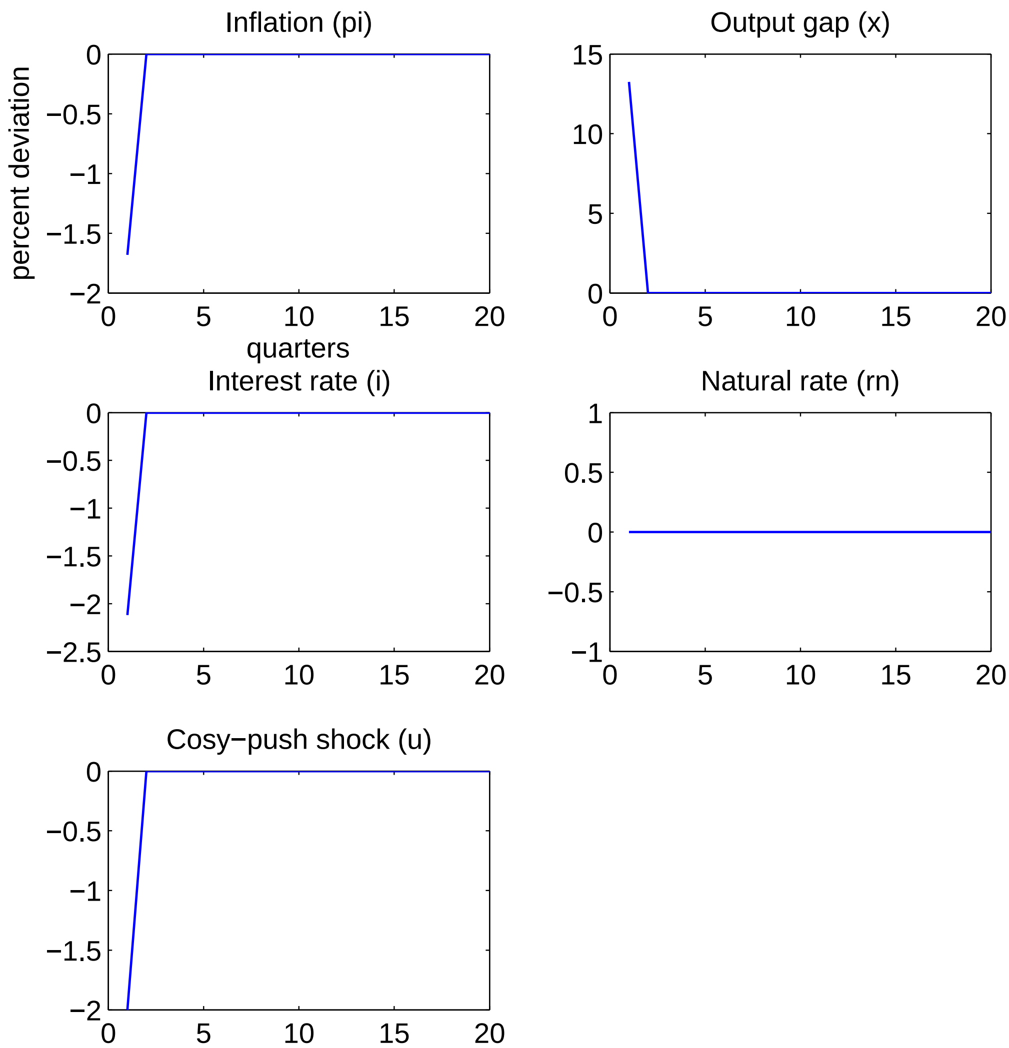

Figure 3: Impulse responses to -2 percent cost-push shock (Discretion)

MATLAB RBC model

```
% RBC model

clear all
close

% Parameters
betta = .988; % discount factor
alfa = .4; % fraction of capital
delta = .025; % depreciation rate
psy  = 0;   % elasticity of labor
pssy = 1;   % fraction of disutility
rho  = .95; % persistence of technology

% calculate steady state
ky = alfa*betta/(1-betta*(1-delta)); % k*/y*
iy = delta*ky;                       % i*/y*
cy = 1-iy;                           % c*/y*
hstar = ((1-alfa)/(pssy*cy))^(1/(1+psy)); % h*
ystar = ky^(alfa/(1-alfa))*hstar; % y*
cstar = cy*ystar;                    % c*
istar = iy*ystar;                    % i*
kstar = ky*ystar;                    % k*

% System of the model:
% A*X(t) = B*X(t-1) + C*eps(t) + D*eta(t)
% X = [c, y, h, i, k, Ec, Ey, a]
% eps(t): Technology shock
% eta(t) = [eta_c,eta_y]: Expectation error

% Prepare matrices

N = 8; neps = 1; neta = 2; % # of variables, shocks, and expectation
    errors.

A = zeros(N,N);
B = zeros(N,N);
C = zeros(N,neps);
D = zeros(N,neta);

% Insert values

% 1. Expectation error of Ec
A(1,1) = 1;
B(1,6) = 1;
D(1,1) = 1;
```

```

% 2. Expectation error of Ey
A(2,2) = 1;
B(2,7) = 1;
D(2,2) = 1;

% 3. Euler equation
A(3,1) = 1;
A(3,5) = -(1-betta*(1-delta));
A(3,6) = -1;
A(3,7) = 1-betta*(1-delta);

% 4. Labor supply
A(4,1) = 1;
A(4,2) = -1;
A(4,3) = 1+psy;

% 5. Product function
A(5,2) = 1;
A(5,3) = -(1-alfa);
A(5,8) = -1;
B(5,5) = alfa;

% 6. Capital accumulation
A(6,4) = -delta;
A(6,5) = 1;
B(6,5) = 1-delta;

% 7. Resource constraint
A(7,1) = -cy;
A(7,2) = 1;
A(7,4) = -iy;

% 8. Technology shock
A(8,8) = 1;
B(8,8) = rho;
C(8,1) = 1;

%
% Eigen value decomposition
%
% Apply generalized schur(QZ) decomposition for S and T.
%
%  S*E[X(t+1)] = T*X(t)
%   (Q'*V1*Z')*E[X(t+1)] = (Q'*V2*Z')*X(t)
%
```

```
% where QQ' = ZZ' = I(× NN).
%
[S,T,Q,Z] = qz(A,B);
s = diag(S);
t = diag(T);
lambda = abs(t./s) < 1-eps;
%
% lambda is generalized eigenvalue;
%   lambda(i) = t(ii)/s(ii)
%   lambda(i) < 1 is stable root.
% Then lambda is a dummy vector which return 1 if eigen value is
    stable.
%
% reorder by recalling ordqz
[S,T,Q,Z] = ordqz(S,T,Q,Z,lambda);
s = diag(S);
t = diag(T);
lambda = abs(t./s) < 1-eps;
NS = sum(lambda);
%
% NS is the number of state variable.
%

% Blanchard and Kahn condition
if NS == N-neta
   disp(' Checking Blanchard and Kahn condition (BKC)... ')
   disp([' ',num2str(N-NS),' unstable roots'])
   disp([' ',num2str(neta),' jump variables'])
   disp([' ',num2str(NS),' stable roots'])
   disp([' ',num2str(N-neta),' state variables'])
   disp(' BKC is satisfied.')
   disp(' ')
else
   disp(' Checking Blanchard and Kahn condition (BKC)...')
   disp([' ',num2str(N-NS),' unstable roots'])
   disp([' ',num2str(neta),' jump variables'])
   disp([' ',num2str(NS),' stable roots'])
   disp([' ',num2str(N-neta),' state variables'])
   error('BKC is not satisfied')
end

% Divide stable and unstable block
%
% Q'*S*Z'*s(t) = Q'*T*Z'*s(t-1)+Psi*eps(t)+Pi*eta(t)
%
% where S and T are upper triangular
```

```
%
S11 = S(1:NS,1:NS);
S12 = S(1:NS,NS+1:N);
S22 = S(NS+1:N,NS+1:N);
T11 = T(1:NS,1:NS);
T12 = T(1:NS,NS+1:N);
T22 = T(NS+1:N,NS+1:N);
%
% Solution form is
%
%  s(t) = M*s(t-1)+Me*eps(t),
%
% where
%
%  M = Z*(S11\T11 S11\(T12-Phi*Tss);0 0)*Z'
%  Me = Z*(S11\(Q1-Phi*Q2);0)*C
%  Phi = Q1*D*((Q2*D)\eye(size(Q2*D,2)));
%
Q1 = Q(1:NS,:); % stable block of Q
Q2 = Q(NS+1:N,:); % unstable block of Q
Phi = Q1*D*((Q2*D)\eye(size(Q2*D,2))); % definition of Phi
GG = zeros(N,N);
GG11 = S11\T11;
GG12 = S11\(T12-Phi*T22);
GG(1:NS,1:NS) = GG11; % insert GG11 into stable block of GG
GG(1:NS,NS+1:N) = GG12; % insert GG12 into unstable block of GG
% Mx
Mx = Z*GG*(Z\eye(size(Z,2)));
GZ = zeros(N,N);
GZ(1:NS,:) = S11\(Q1-Phi*Q2);
% Me
Me = Z*GZ*C;

%
% Impulse responses
%

T = 20; % irf period
X = zeros(N,T); % box
epsa = 1;

% for initial period
X(:,1) = Me*epsa';
% following periods
for t = 2:T
   X(:,t) = Mx*X(:,t-1);
```

```
end
%Extradct and set names
c = X(1,:);
y = X(2,:);
n = X(3,:);
i = X(4,:);
k = X(5,:);
Ec = X(6,:);
Ey = X(7,:);
a = X(8,:);


% Plot impulse responses
%
% Figure plot
%
lw = 1;              % line width of impulse responses
zeroline = zeros(1,T); % steady state lines

figure(1)

subplot(3,2,1) % output
 h = plot(1:T,y,1:T,zeroline); % plot(irf period, irf, irf period, zeroline)
 set(h(1),'linewidth',lw) % irf line width
 set(h(2),'color','red') % zeroline color (set red in this time)
 title('Output (y)')    % irf name
 xlabel('quarters')     % label name of x axis
 ylabel('percent deviation') % label name of y axis

subplot(3,2,2) % consumption
 h = plot(1:T,c,1:T,zeroline);
 set(h(1),'linewidth',lw)
 set(h(2),'color','red')
 title('Consumption (c)')

subplot(3,2,3) % investment
 h = plot(1:T,i,1:T,zeroline);
 set(h(1),'linewidth',lw)
 set(h(2),'color','red')
 title('Investment (i)')

subplot(3,2,4) % labor
 h = plot(1:T,n,1:T,zeroline);
 set(h(1),'linewidth',lw)
 set(h(2),'color','red')
```

```
 title('Labor (h)')

subplot(3,2,5) % technology
 h = plot(1:T,a,1:T,zeroline);
 set(h(1),'linewidth',lw)
 set(h(2),'color','red')
 title('Technology (a)')
```

MATLAB Code: Optimal Monetary Policy (Commitment)

```
% Optimal monetary policy, Commitment
%
% NK model
%
% period loss: 1/2*(pi^2 + x^2)
%
% strucutural equations:
% x = Ex - sig*(i - Epi -rn)
% p = beta*Ep + kpp*x + u
% rn = rhor*rn(-1) + epsrn
% u = rhou*rn(-1) + epsu
%
% FOC
%  p = phip - phip(-1)
%  x = -kappa/lambdax*phip

clear all
close

% Parameters
betta = .99; % discount factor
sig   = 6.25; % intertemporal elastisity
kappa = 0.024; % slope of NKPC
tht   = 7.88; % depreciation rate
lambdax = kappa/tht; % weight on output gap
rhor  = .5;  % persistence of natural rate
rhou  = 0; % persistence of cost shock

% System of the model:
% A*X(t) = B*X(t-1) + C*eps(t) + D*eta(t)
% X = [pi, x, i, phip, Ex, Epi, rn, u]
% eps(t): Technology shock
% eta(t) = [eta_c,eta_y]: Expectation error

% Prepare matrices

N = 8; neps = 2; neta = 2; % # of variables, shocks, and expectation
    errors.

A = zeros(N,N);
B = zeros(N,N);
C = zeros(N,neps);
D = zeros(N,neta);

% 1. NKPC
```

```
A(1,1) = 1;
A(1,2) = -kappa;
A(1,6) = -betta;
A(1,8) = -1;

% 2. DIS
A(2,2) = 1;
A(2,3) = sig;
A(2,5) = -1;
A(2,6) = -sig;
A(2,7) = -sig;

% 3. Foc for inflation
A(3,1) = 1;
A(3,4) = -1;
B(3,4) = -1;

% 4. Foc for output gap
A(4,2) = 1;
A(4,4) = kappa/lambdax;

% 5. AR1 for rn
A(5,7) = 1;
B(5,7) = rhor;
C(5,1) = 1;

% 6. AR1 for u
A(6,8) = 1;
B(6,8) = rhou;
C(6,2) = 1;

% 7. Expectation error of inflation
A(7,1) = 1;
B(7,6) = 1;
D(7,2) = 1;

% 8. Expectation error of inflation
A(8,2) = 1;
B(8,5) = 1;
D(8,1) = 1;

%
% Eigen value decomposition
%
% Apply generalized schur(QZ) decomposition for S and T.
%
```

```
%  S*E[X(t+1)] = T*X(t)
%   (Q'*V1*Z')*E[X(t+1)] = (Q'*V2*Z')*X(t)
%
% where QQ' = ZZ' = I(× NN).
%
[S,T,Q,Z] = qz(A,B);
s = diag(S);
t = diag(T);
lambda = abs(t./s) < 1-eps;
%
% lambda is generalized eigenvalue;
%   lambda(i) = t(ii)/s(ii)
%   lambda(i) < 1 is stable root.
% Then lambda is a dummy vector which return 1 if eigen value is
   stable.
%
% reorder by recalling ordqz
[S,T,Q,Z] = ordqz(S,T,Q,Z,lambda);
s = diag(S);
t = diag(T);
lambda = abs(t./s) < 1-eps;
NS = sum(lambda);
%
% NS is the number of state variable.
%

% Blanchard and Kahn condition
if NS == N-neta
   disp(' Checking Blanchard and Kahn condition (BKC)... ')
   disp([' ',num2str(N-NS),' unstable roots'])
   disp([' ',num2str(neta),' jump variables'])
   disp([' ',num2str(NS),' stable roots'])
   disp([' ',num2str(N-neta),' state variables'])
   disp(' BKC is satisfied.')
   disp(' ')
else
   disp(' Checking Blanchard and Kahn condition (BKC)...')
   disp([' ',num2str(N-NS),' unstable roots'])
   disp([' ',num2str(neta),' jump variables'])
   disp([' ',num2str(NS),' stable roots'])
   disp([' ',num2str(N-neta),' state variables'])
   error('BKC is not satisfied')
end

% Divide stable and unstable block
%
```

```
% Q'*S*Z'*s(t) = Q'*T*Z'*s(t-1)+Psi*eps(t)+Pi*eta(t)
%
% where S and T are upper triangular
%
S11 = S(1:NS,1:NS);
S12 = S(1:NS,NS+1:N);
S22 = S(NS+1:N,NS+1:N);
T11 = T(1:NS,1:NS);
T12 = T(1:NS,NS+1:N);
T22 = T(NS+1:N,NS+1:N);
%
% Solution form is
%
%  s(t) = M*s(t-1)+Me*eps(t),
%
% where
%
%  M = Z*(S11\T11 S11\(T12-Phi*Tss);0 0)*Z'
%  Me = Z*(S11\(Q1-Phi*Q2);0)*C
%  Phi = Q1*D*((Q2*D)\eye(size(Q2*D,2)));
%
Q1 = Q(1:NS,:); % stable block of Q
Q2 = Q(NS+1:N,:); % unstable block of Q
Phi = Q1*D*((Q2*D)\eye(size(Q2*D,2))); % definition of Phi
GG = zeros(N,N);
GG11 = S11\T11;
GG12 = S11\(T12-Phi*T22);
GG(1:NS,1:NS) = GG11; % insert GG11 into stable block of GG
GG(1:NS,NS+1:N) = GG12; % insert GG12 into unstable block of GG
% Mx
Mx = Z*GG*(Z\eye(size(Z,2)));
GZ = zeros(N,N);
GZ(1:NS,:) = S11\(Q1-Phi*Q2);
% Me
Me = Z*GZ*C;

%
% Impulse responses
%

T = 20; % irf period
X = zeros(N,T); % box

varrn = 0; % natural rate shock
varu = -2; % cost-push shock

```

```
eps_t = [varrn,varu];

% for initial period
X(:,1) = Me*eps_t';
% following periods
for t = 2:T
   X(:,t) = Mx*X(:,t-1);
end
%Extradct and set names
p = X(1,:);
x = X(2,:);
i = X(3,:);
rn = X(7,:);
u = X(8,:);


% Plot impulse responses
%
% Figure plot
%
lw = 1;              % line width of impulse responses
zeroline = zeros(1,T); % steady state lines

figure(1)

subplot(3,2,1) % output
 h = plot(1:T,p); % plot(irf period, irf, irf period, zeroline)
 set(h(1),'linewidth',lw) % irf line width
 title('Inflation (\pi)')  % irf name
 xlabel('quarters')    % label name of x axis
 ylabel('percent deviation') % label name of y axis

subplot(3,2,2) % consumption
 h = plot(1:T,x);
 set(h(1),'linewidth',lw)
 title('Output gap (x)')

subplot(3,2,3) % investment
 h = plot(1:T,i);
 set(h(1),'linewidth',lw)
 title('Interest rate (i)')

subplot(3,2,4) % labor
 h = plot(1:T,rn);
 set(h(1),'linewidth',lw)
 title('Natural rate (r^n)')
```

```

subplot(3,2,5) % technology
 h = plot(1:T,u);
 set(h(1),'linewidth',lw)
 title('Cosy-push shock (u)')
```

MATLAB Code: Optimal Monetary Policy (Discretion)

```
% Optimal monetary policy, Discretion
%
% NK model
%
% period loss: 1/2*(pi^2 + x^2)
%
% strucutural equations:
% x = Ex - sig*(i - Epi -rn)
% p = beta*Ep + kpp*x + u
% rn = rhor*rn(-1) + epsrn
% u = rhou*rn(-1) + epsu
%
% FOC
%  p = phip
%  x = -kappa/lambdax*phip


clear all
close

% Parameters
betta = .99; % discount factor
sig   = 6.25; % intertemporal elastisity
kappa = 0.024; % slope of NKPC
tht   = 7.88; % depreciation rate
lambdax = kappa/tht; % weight on output gap
rhor  = .5;  % persistence of natural rate
rhou  = 0; % persistence of cost shock

% System of the model:
% A*X(t) = B*X(t-1) + C*eps(t) + D*eta(t)
% X = [pi, x, i, phip, Ex, Epi, rn, u]
% eps(t): Technology shock
% eta(t) = [eta_c,eta_y]: Expectation error

% Prepare matrices

N = 8; neps = 2; neta = 2; % # of variables, shocks, and expectation
    errors.

A = zeros(N,N);
B = zeros(N,N);
C = zeros(N,neps);
D = zeros(N,neta);

```

```
% 1. NKPC
A(1,1) = 1;
A(1,2) = -kappa;
A(1,6) = -betta;
A(1,8) = -1;

% 2. DIS
A(2,2) = 1;
A(2,3) = sig;
A(2,5) = -1;
A(2,6) = -sig;
A(2,7) = -sig;

% 3. Foc for inflation
A(3,1) = 1;
A(3,4) = -1;

% 4. Foc for output gap
A(4,2) = 1;
A(4,4) = kappa/lambdax;

% 5. AR1 for rn
A(5,7) = 1;
B(5,7) = rhor;
C(5,1) = 1;

% 6. AR1 for u
A(6,8) = 1;
B(6,8) = rhou;
C(6,2) = 1;

% 7. Expectation error of inflation
A(7,1) = 1;
B(7,6) = 1;
D(7,2) = 1;

% 8. Expectation error of inflation
A(8,2) = 1;
B(8,5) = 1;
D(8,1) = 1;

%
% Eigen value decomposition
%
% Apply generalized schur(QZ) decomposition for S and T.
%
```

```
%  S*E[X(t+1)] = T*X(t)
%   (Q'*V1*Z')*E[X(t+1)] = (Q'*V2*Z')*X(t)
%
%  where QQ' = ZZ' = I(× NN).
%
[S,T,Q,Z] = qz(A,B);
s = diag(S);
t = diag(T);
lambda = abs(t./s) < 1-eps;
%
% lambda is generalized eigenvalue;
%   lambda(i) = t(ii)/s(ii)
%   lambda(i) < 1 is stable root.
% Then lambda is a dummy vector which return 1 if eigen value is stable.
%
% reorder by recalling ordqz
[S,T,Q,Z] = ordqz(S,T,Q,Z,lambda);
s = diag(S);
t = diag(T);
lambda = abs(t./s) < 1-eps;
NS = sum(lambda);
%
% NS is the number of state variable.
%

% Blanchard and Kahn condition
if NS == N-neta
   disp(' Checking Blanchard and Kahn condition (BKC)... ')
   disp([' ',num2str(N-NS),' unstable roots'])
   disp([' ',num2str(neta),' jump variables'])
   disp([' ',num2str(NS),' stable roots'])
   disp([' ',num2str(N-neta),' state variables'])
   disp(' BKC is satisfied.')
   disp(' ')
else
   disp(' Checking Blanchard and Kahn condition (BKC)...')
   disp([' ',num2str(N-NS),' unstable roots'])
   disp([' ',num2str(neta),' jump variables'])
   disp([' ',num2str(NS),' stable roots'])
   disp([' ',num2str(N-neta),' state variables'])
   error('BKC is not satisfied')
end

% Divide stable and unstable block
%
```

```
% Q'*S*Z'*s(t) = Q'*T*Z'*s(t-1)+Psi*eps(t)+Pi*eta(t)
%
% where S and T are upper triangular
%
S11 = S(1:NS,1:NS);
S12 = S(1:NS,NS+1:N);
S22 = S(NS+1:N,NS+1:N);
T11 = T(1:NS,1:NS);
T12 = T(1:NS,NS+1:N);
T22 = T(NS+1:N,NS+1:N);
%
% Solution form is
%
%  s(t) = M*s(t-1)+Me*eps(t),
%
% where
%
%  M = Z*(S11\T11 S11\(T12-Phi*Tss);0 0)*Z'
%  Me = Z*(S11\(Q1-Phi*Q2);0)*C
%  Phi = Q1*D*((Q2*D)\eye(size(Q2*D,2)));
%
Q1 = Q(1:NS,:); % stable block of Q
Q2 = Q(NS+1:N,:); % unstable block of Q
Phi = Q1*D*((Q2*D)\eye(size(Q2*D,2))); % definition of Phi
GG = zeros(N,N);
GG11 = S11\T11;
GG12 = S11\(T12-Phi*T22);
GG(1:NS,1:NS) = GG11; % insert GG11 into stable block of GG
GG(1:NS,NS+1:N) = GG12; % insert GG12 into unstable block of GG
% Mx
Mx = Z*GG*(Z\eye(size(Z,2)));
GZ = zeros(N,N);
GZ(1:NS,:) = S11\(Q1-Phi*Q2);
% Me
Me = Z*GZ*C;

%
% Impulse responses
%

T = 20; % irf period
X = zeros(N,T); % box

varrn = 0; % natural rate shock
varu = -2; % cost-push shock

```

```
eps_t = [varrn,varu];

% for initial period
X(:,1) = Me*eps_t';
% following periods
for t = 2:T
   X(:,t) = Mx*X(:,t-1);
end
%Extradct and set names
p = X(1,:);
x = X(2,:);
i = X(3,:);
rn = X(7,:);
u = X(8,:);


% Plot impulse responses
%
% Figure plot
%
lw = 1;              % line width of impulse responses
zeroline = zeros(1,T); % steady state lines

figure(1)

subplot(3,2,1) % output
 h = plot(1:T,p); % plot(irf period, irf, irf period, zeroline)
 set(h(1),'linewidth',lw) % irf line width
 title('Inflation (pi)')   % irf name
 xlabel('quarters')     % label name of x axis
 ylabel('percent deviation') % label name of y axis

subplot(3,2,2) % consumption
 h = plot(1:T,x);
 set(h(1),'linewidth',lw)
 title('Output gap (x)')

subplot(3,2,3) % investment
 h = plot(1:T,i);
 set(h(1),'linewidth',lw)
 title('Interest rate (i)')

subplot(3,2,4) % labor
 h = plot(1:T,rn);
 set(h(1),'linewidth',lw)
 title('Natural rate (rn)')
```

```

subplot(3,2,5) % technology
 h = plot(1:T,u);
 set(h(1),'linewidth',lw)
 title('Cosy-push shock (u)')
```