

演習マクロ経済学 第14回 講義資料

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本ハンドアウトに掲載されている MATLAB コードは <http://khas.bitbucket.org/> からダウンロード可能

1 RBC model

1.1 モデル

$$\max_{c_t, k_{t+1}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\log c_t - \Psi \frac{h_t^{1+\psi}}{1+\psi} \right),$$

$$\text{s.t. } y_t = c_t + k_{t+1} - (1 - \delta)k_t,$$

$$y_t = a_t k_t^\alpha h_t^{1-\alpha},$$

$$\lim_{t \rightarrow \infty} \{\beta^t \lambda_t k_{t+1}\} = 0,$$

$$a_t = (1 - \rho)\bar{a} + \rho a_{t-1} + \epsilon_t$$

F.O.Cs :

$$1 = \beta \mathbb{E}_t \left[\frac{c_t}{c_{t+1}} \left(\alpha \frac{y_{t+1}}{k_{t+1}} + 1 - \delta \right) \right]$$

$$\Psi h_t^\psi c_t = (1 - \alpha) \frac{y_t}{h_t}$$

1.2 定常状態

$$1 = \beta \left(\alpha \frac{y^*}{k^*} + 1 - \delta \right),$$

$$\Psi (h^*)^\psi c^* = (1 - \alpha) \frac{y^*}{h^*}$$

$$y^* = \bar{a} (k^*)^\alpha (h^*)^{1-\alpha},$$

$$y^* = c^* + i^*,$$

$$k^* = i^* + (1 - \delta)k^*$$

定常状態の計算

$$\begin{aligned} \frac{k^*}{y^*} &= \frac{\alpha\beta}{1-\beta(1-\delta)} \rightarrow \frac{i^*}{y^*} = \delta \frac{k^*}{y^*} \rightarrow \frac{c^*}{y^*} = 1 - \frac{i^*}{y^*} \rightarrow h^* = \left(\frac{1-\alpha}{\Psi(c^*/y^*)} \right)^{\frac{1}{1+\psi}}, \\ \rightarrow y^* &= \bar{a} \left(\frac{k^*}{y^*} \right)^{\frac{\alpha}{1-\alpha}} h^* \rightarrow c^* = \left(\frac{c^*}{y^*} \right) y^* \rightarrow i^* = \left(\frac{i^*}{y^*} \right) y^* \end{aligned}$$

1.3 モデル（線形化後）

以下の変数は定常状態からの乖離率を表す．

$$c_t = \mathbb{E}_{t-1} c_t + \eta_t^c, \quad (1)$$

$$y_t = \mathbb{E}_{t-1} y_t + \eta_t^y, \quad (2)$$

$$c_t - \mathbb{E} c_{t+1} + [1 - \beta(1 - \delta)] (\mathbb{E} y_{t+1} - k_{t+1}) = 0, \quad (3)$$

$$(1 + \psi) h_t + c_t - y_t = 0, \quad (4)$$

$$y_t - (1 - \alpha) h_t - a_t = \alpha h_t, \quad (5)$$

$$k_{t+1} - \delta k_t = (1 - \alpha) k_t, \quad (6)$$

$$y_t - \frac{c^*}{y^*} c_t - \frac{i^*}{y^*} i_t = 0, \quad (7)$$

$$a_t = \rho a_{t-1} + \epsilon_t \quad (8)$$

行列表記：

$$AX_t = BX_{t-1} + C\epsilon_t + D\eta_t,$$

$$X_t = \begin{bmatrix} c_t & y_t & h_t & i_t & k_t & \mathbb{E}_t c_{t+1} & \mathbb{E}_t y_{t+1} & a_t \end{bmatrix}^\top$$

$$\eta_t = \begin{bmatrix} \eta_t^c & \eta_t^y \end{bmatrix}^\top$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & -[1 - \beta(1 - \delta)] & -1 & [1 - \beta(1 - \delta)] & 0 \\ 1 & -1 & (1 + \psi) & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & (1 - \alpha) & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & -\delta & 1 & 0 & 0 & 0 \\ -c^*/y^* & 1 & 0 & -i^*/y^* & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \alpha & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 - \delta & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho \end{bmatrix}, \quad C = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$

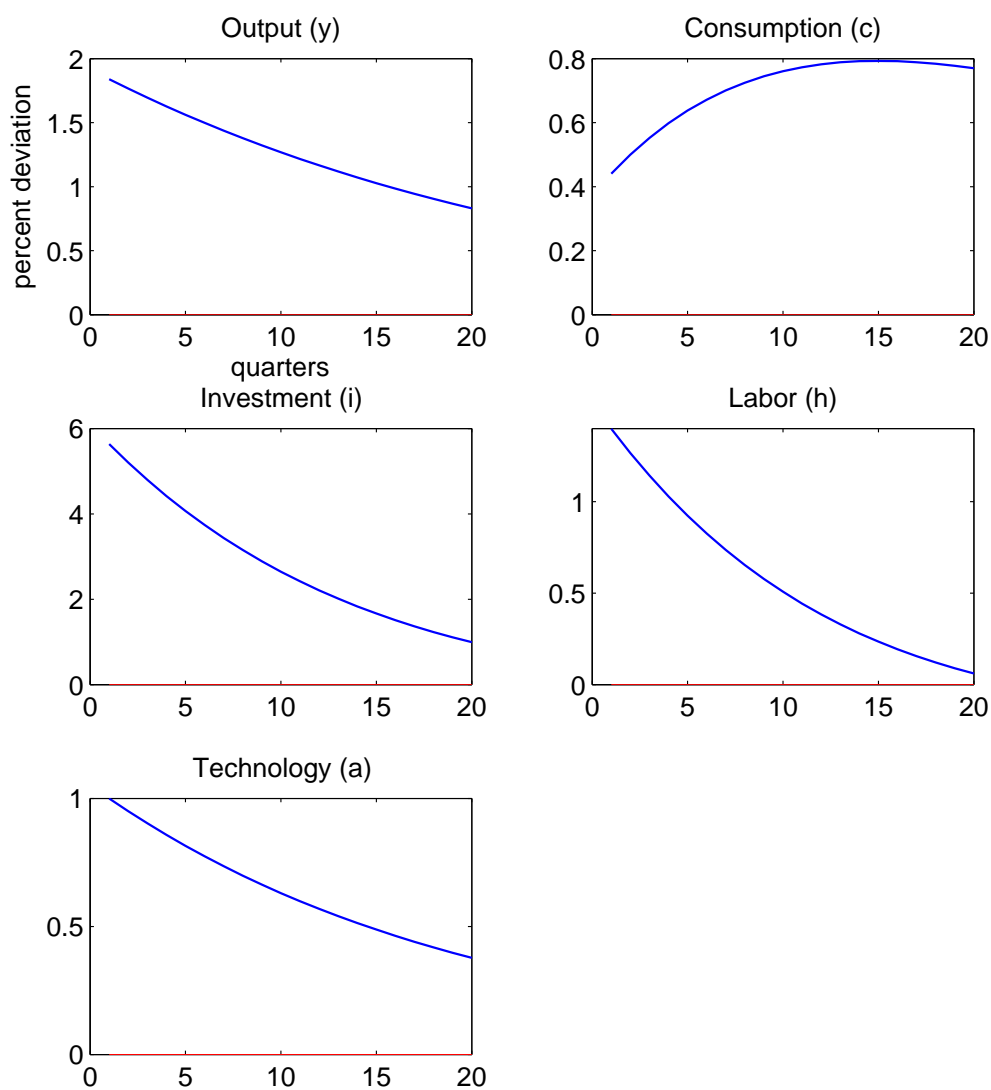


Figure 1: Impulse responses to 1 percent technology shock

2 Optimal Monetary Policy

$$\begin{aligned}
& \min_{\pi_t, x_t, i_t} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \frac{1}{2} [\pi_t^2 + \lambda_x x_t^2], \\
& \text{s.t.} \quad \pi_t = \beta \mathbb{E}_t \pi_{t+1} + \kappa x_t + u_t, \\
& \quad x_t = \mathbb{E}_t x_{t+1} - \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n), \\
& \quad r_t^n = \rho_r r_{t-1}^n + \epsilon_t^r, \\
& \quad u_t = \rho_u u_{t-1} + \epsilon_t^u
\end{aligned}$$

2.1 Commitment

F.O.Cs:

$$\begin{aligned}
\pi_t - \phi_{\pi t} &= \phi_{\pi t-1}, \\
x_t + \frac{\kappa}{\lambda_x} \phi_{\pi t} &= 0,
\end{aligned}$$

2.2 Discretion

F.O.Cs:

$$\begin{aligned}
\pi_t - \phi_{\pi t} &= 0, \\
x_t + \frac{\kappa}{\lambda_x} \phi_{\pi t} &= 0,
\end{aligned}$$

2.3 System of the model (Commitment)

$$\begin{aligned}
\pi_t - \beta \mathbb{E}_t \pi_{t+1} - \kappa x_t - u_t &= 0, \\
x_t - \mathbb{E}_t x_{t+1} + \sigma(i_t - \mathbb{E}_t \pi_{t+1} - r_t^n) &= 0, \\
\pi_t - \phi_{\pi t} &= \phi_{\pi t-1}, \\
x_t + \frac{\kappa}{\lambda_x} \phi_{\pi t} &= 0, \\
r_t^n &= \rho_r r_{t-1}^n + \epsilon_t^r, \\
u_t &= \rho_u u_{t-1} + \epsilon_t^u, \\
\pi_t &= \mathbb{E}_{t-1} \pi_t + \eta_t^\pi, \\
x_t &= \mathbb{E}_{t-1} x_t + \eta_t^x
\end{aligned}$$

行列表記：

$$AX_t = BX_{t-1} + C\epsilon_t + D\eta_t,$$

$$X_t = \begin{bmatrix} \pi_t & x_t & i_t & \phi_{\pi t} & \mathbb{E}_t x_{t+1} & \mathbb{E}_t \pi_{t+1} & r_t^n & u_t \end{bmatrix}^\top$$

$$\eta_t = \begin{bmatrix} \eta_t^\pi & \eta_t^x \end{bmatrix}^\top$$

$$\epsilon_t = \begin{bmatrix} \epsilon_t^r & \epsilon_t^u \end{bmatrix}^\top$$

$$A = \begin{bmatrix} 1 & -\kappa & 0 & 0 & 0 & -\beta & 0 & -1 \\ 0 & 1 & \sigma & 0 & -1 & -\sigma & -\sigma & 0 \\ 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & \kappa/\lambda_x & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \rho_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \rho_u \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \quad D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 1 \\ 1 & 0 \end{bmatrix}$$

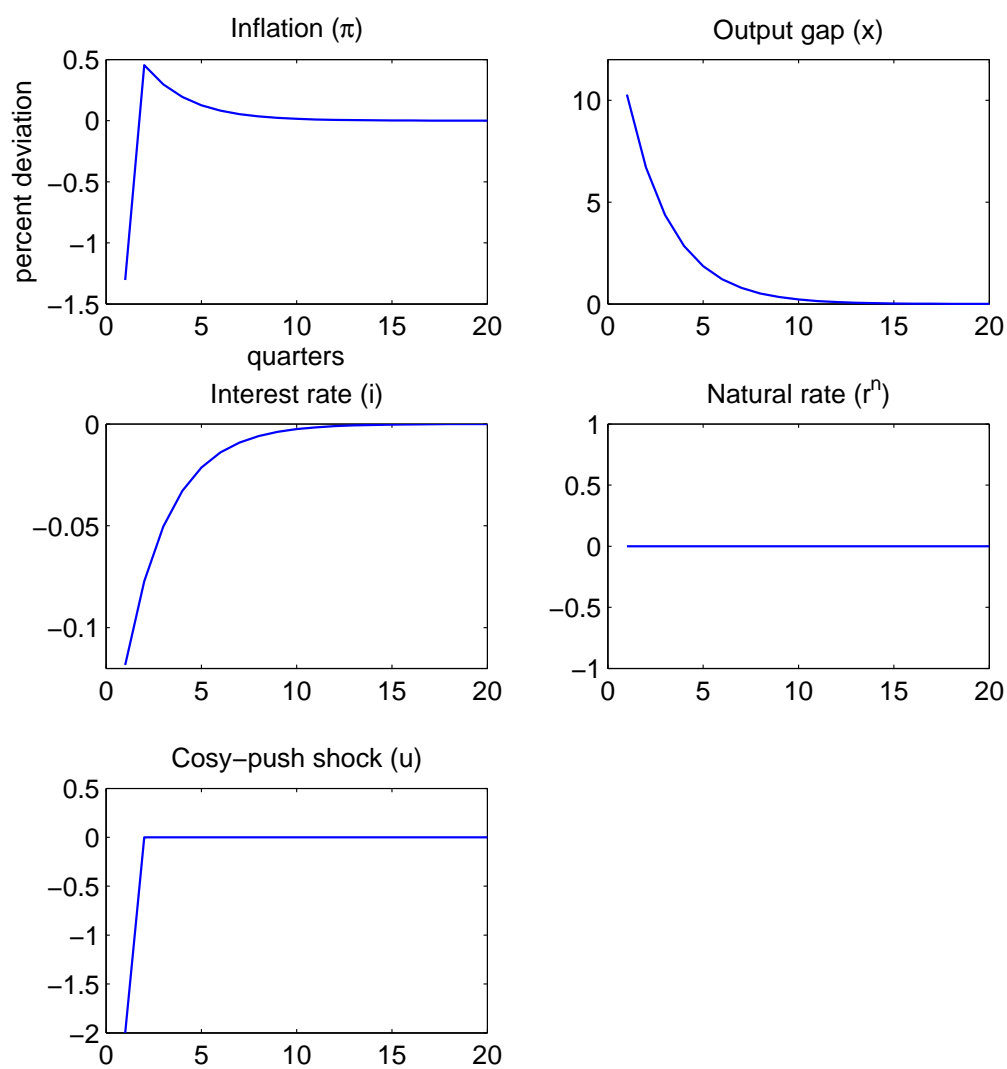


Figure 2: Impulse responses to -2 percent cost-push shock (Commitment)

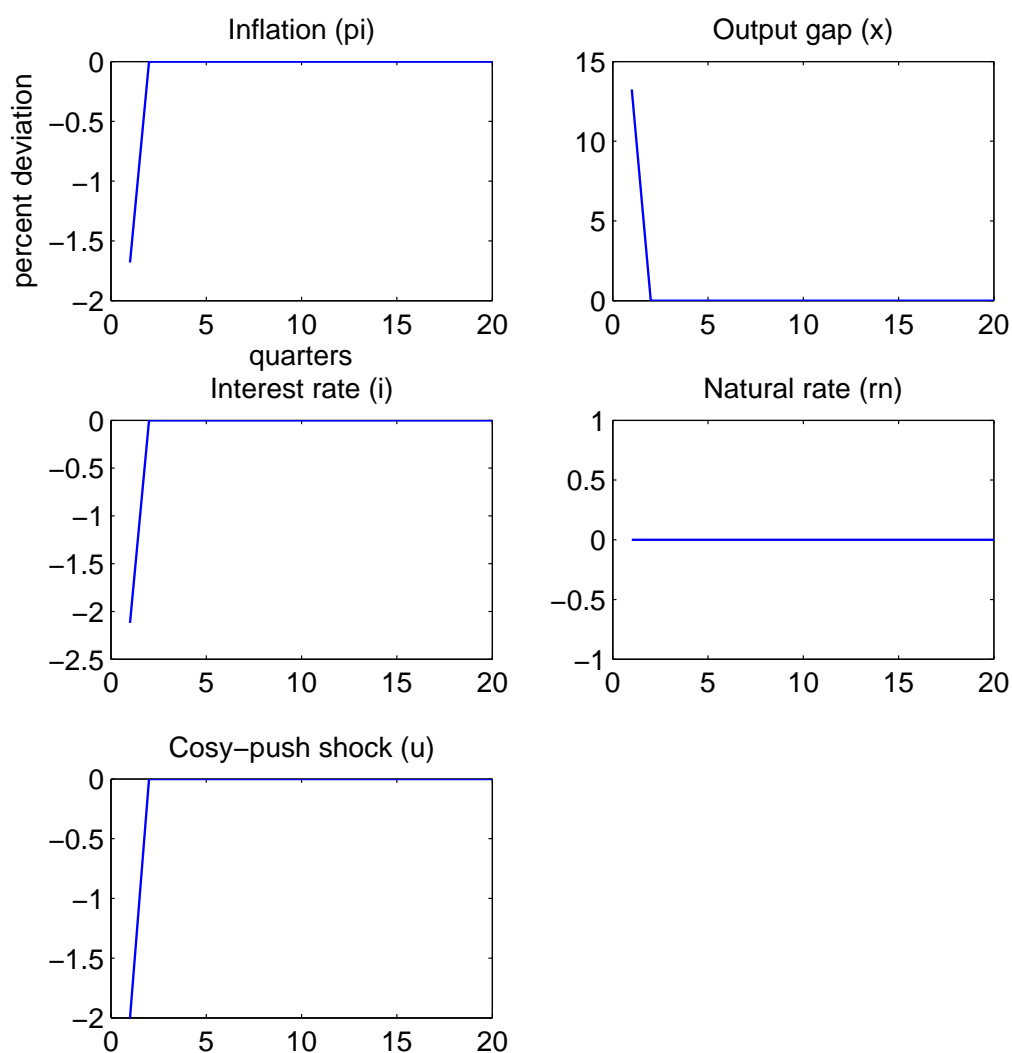


Figure 3: Impulse responses to -2 percent cost-push shock (Discretion)

MATLAB RBC model

```

1 % RBC model
2
3 clear all
4 close
5
6 % Parameters
7 betta = .988; % discount factor
8 alfa = .4; % fraction of capital
9 delta = .025; % depreciation rate
10 psy = 0; % elasticity of labor
11 pssy = 1; % fraction of disutility
12 rho = .95; % persistence of technology
13
14 % calculate steady state
15 ky = alfa*betta/(1-betta*(1-delta)); % k*/y*
16 iy = delta*ky; % i*/y*
17 cy = 1-iy; % c*/y*
18 hstar = ((1-alfa)/(pssy*cy))^(1/(1+psy)); % h*
19 ystar = ky^(alfa/(1-alfa))*hstar; % y*
20 cstar = cy*ystar; % c*
21 istar = iy*ystar; % i*
22 kstar = ky*ystar; % k*
23
24 % System of the model:
25 % A*X(t) = B*X(t-1) + C*eps(t) + D*eta(t)
26 % X = [c, y, h, i, k, Ec, Ey, a]
27 % eps(t): Technology shock
28 % eta(t) = [eta_c, eta_y]: Expectation error
29
30 % Prepare matrices
31
32 N = 8; neps = 1; neta = 2; % # of variables, shocks, and expectation
    errors.
33
34 A = zeros(N,N);
35 B = zeros(N,N);
36 C = zeros(N,neps);
37 D = zeros(N,neta);
38
39 % Insert values
40
41 % 1. Expectation error of Ec
42 A(1,1) = 1;
43 B(1,6) = 1;
44 D(1,1) = 1;

```

```

45
46 % 2. Expectation error of Ey
47 A(2,2) = 1;
48 B(2,7) = 1;
49 D(2,2) = 1;
50
51 % 3. Euler equation
52 A(3,1) = 1;
53 A(3,5) = -(1-betta*(1-delta));
54 A(3,6) = -1;
55 A(3,7) = 1-betta*(1-delta);
56
57 % 4. Labor supply
58 A(4,1) = 1;
59 A(4,2) = -1;
60 A(4,3) = 1+psy;
61
62 % 5. Product function
63 A(5,2) = 1;
64 A(5,3) = -(1-alfa);
65 A(5,8) = -1;
66 B(5,5) = alfa;
67
68 % 6. Capital accumulation
69 A(6,4) = -delta;
70 A(6,5) = 1;
71 B(6,5) = 1-delta;
72
73 % 7. Resource constraint
74 A(7,1) = -cy;
75 A(7,2) = 1;
76 A(7,4) = -iy;
77
78 % 8. Technology shock
79 A(8,8) = 1;
80 B(8,8) = rho;
81 C(8,1) = 1;
82
83 %
84 % Eigen value decomposition
85 %
86 % Apply generalized schur(QZ) decomposition for S and T.
87 %
88 %  $S \cdot E[X(t+1)] = T \cdot X(t)$ 
89 %  $(Q' \cdot V1 \cdot Z') \cdot E[X(t+1)] = (Q' \cdot V2 \cdot Z') \cdot X(t)$ 
90 %

```

```

91 % where  $QQ' = ZZ' = I(\times NN)$ .
92 %
93 [S,T,Q,Z] = qz(A,B);
94 s = diag(S);
95 t = diag(T);
96 lambda = abs(t./s) < 1-eps;
97 %
98 % lambda is generalized eigenvalue;
99 % lambda(i) = t(ii)/s(ii)
100 % lambda(i) < 1 is stable root.
101 % Then lambda is a dummy vector which return 1 if eigen value is
    stable.
102 %
103 % reorder by recalling ordqz
104 [S,T,Q,Z] = ordqz(S,T,Q,Z,lambda);
105 s = diag(S);
106 t = diag(T);
107 lambda = abs(t./s) < 1-eps;
108 NS = sum(lambda);
109 %
110 % NS is the number of state variable.
111 %
112
113 % Blanchard and Kahn condition
114 if NS == N-neta
115     disp(' Checking Blanchard and Kahn condition (BKC)... ')
116     disp([' ',num2str(N-NS),' unstable roots'])
117     disp([' ',num2str(neta),' jump variables'])
118     disp([' ',num2str(NS),' stable roots'])
119     disp([' ',num2str(N-neta),' state variables'])
120     disp(' BKC is satisfied.')
121     disp(' ')
122 else
123     disp(' Checking Blanchard and Kahn condition (BKC)...')
124     disp([' ',num2str(N-NS),' unstable roots'])
125     disp([' ',num2str(neta),' jump variables'])
126     disp([' ',num2str(NS),' stable roots'])
127     disp([' ',num2str(N-neta),' state variables'])
128     error('BKC is not satisfied')
129 end
130
131 % Divide stable and unstable block
132 %
133 %  $Q'S*Z'*s(t) = Q'*T*Z'*s(t-1)+Psi*eps(t)+Pi*eta(t)$ 
134 %
135 % where S and T are upper triangular

```

```

136 %
137 S11 = S(1:NS,1:NS);
138 S12 = S(1:NS,NS+1:N);
139 S22 = S(NS+1:N,NS+1:N);
140 T11 = T(1:NS,1:NS);
141 T12 = T(1:NS,NS+1:N);
142 T22 = T(NS+1:N,NS+1:N);
143 %
144 % Solution form is
145 %
146 %  $s(t) = M*s(t-1) + Me*eps(t)$ ,
147 %
148 % where
149 %
150 %  $M = Z*(S11 \backslash T11 \quad S11 \backslash (T12 - \Phi * T22); 0 \quad 0) * Z'$ 
151 %  $Me = Z*(S11 \backslash (Q1 - \Phi * Q2); 0) * C$ 
152 %  $\Phi = Q1 * D * ((Q2 * D) \backslash eye(size(Q2 * D, 2)))$ ;
153 %
154 Q1 = Q(1:NS,:); % stable block of Q
155 Q2 = Q(NS+1:N,:); % unstable block of Q
156 Phi = Q1 * D * ((Q2 * D) \ eye(size(Q2 * D, 2))); % definition of Phi
157 GG = zeros(N,N);
158 GG11 = S11 \ T11;
159 GG12 = S11 \ (T12 - Phi * T22);
160 GG(1:NS,1:NS) = GG11; % insert GG11 into stable block of GG
161 GG(1:NS,NS+1:N) = GG12; % insert GG12 into unstable block of GG
162 % Mx
163 Mx = Z * GG * (Z \ eye(size(Z, 2)));
164 GZ = zeros(N,N);
165 GZ(1:NS,:) = S11 \ (Q1 - Phi * Q2);
166 % Me
167 Me = Z * GZ * C;
168
169 %
170 % Impulse responses
171 %
172
173 T = 20; % irf period
174 X = zeros(N,T); % box
175 epsa = 1;
176
177 % for initial period
178 X(:,1) = Me * epsa';
179 % following periods
180 for t = 2:T
181     X(:,t) = Mx * X(:,t-1);

```

```

182 end
183 %Extradct and set names
184 c = X(1,:);
185 y = X(2,:);
186 n = X(3,:);
187 i = X(4,:);
188 k = X(5,:);
189 Ec = X(6,:);
190 Ey = X(7,:);
191 a = X(8,:);
192
193
194 % Plot impulse responses
195 %
196 % Figure plot
197 %
198 lw = 1; % line width of impulse responses
199 zeroline = zeros(1,T); % steady state lines
200
201 figure(1)
202
203 subplot(3,2,1) % output
204 h = plot(1:T,y,1:T,zeroline); % plot(irf period, irf, irf period,
    zeroline)
205 set(h(1),'linewidth',lw) % irf line width
206 set(h(2),'color','red') % zeroline color (set red in this time)
207 title('Output (y)') % irf name
208 xlabel('quarters') % label name of x axis
209 ylabel('percent deviation') % label name of y axis
210
211 subplot(3,2,2) % consumption
212 h = plot(1:T,c,1:T,zeroline);
213 set(h(1),'linewidth',lw)
214 set(h(2),'color','red')
215 title('Consumption (c)')
216
217 subplot(3,2,3) % investment
218 h = plot(1:T,i,1:T,zeroline);
219 set(h(1),'linewidth',lw)
220 set(h(2),'color','red')
221 title('Investment (i)')
222
223 subplot(3,2,4) % labor
224 h = plot(1:T,n,1:T,zeroline);
225 set(h(1),'linewidth',lw)
226 set(h(2),'color','red')

```

```
227 title('Labor (h)')
228
229 subplot(3,2,5) % technology
230 h = plot(1:T,a,1:T,zeroline);
231 set(h(1),'linewidth',lw)
232 set(h(2),'color','red')
233 title('Technology (a)')
```

MATLAB Code: Optimal Monetary Policy (Commitment)

```

1 % Optimal monetary policy, Commitment
2 %
3 % NK model
4 %
5 % period loss: 1/2*(pi^2 + x^2)
6 %
7 % structural equations:
8 % x = Ex - sig*(i - Epi -rn)
9 % p = beta*Ep + kpp*x + u
10 % rn = rhor*rn(-1) + epsrn
11 % u = rhou*rn(-1) + epsu
12 %
13 % FOC
14 % p = phip - phip(-1)
15 % x = -kappa/lambdax*phip
16
17 clear all
18 close
19
20 % Parameters
21 betta = .99; % discount factor
22 sig = 6.25; % intertemporal elasticity
23 kappa = 0.024; % slope of NKPC
24 tht = 7.88; % depreciation rate
25 lambdax = kappa/tht; % weight on output gap
26 rhor = .5; % persistence of natural rate
27 rhou = 0; % persistence of cost shock
28
29 % System of the model:
30 % A*X(t) = B*X(t-1) + C*eps(t) + D*eta(t)
31 % X = [pi, x, i, phip, Ex, Epi, rn, u]
32 % eps(t): Technology shock
33 % eta(t) = [eta_c, eta_y]: Expectation error
34
35 % Prepare matrices
36
37 N = 8; neps = 2; neta = 2; % # of variables, shocks, and expectation
    errors.
38
39 A = zeros(N,N);
40 B = zeros(N,N);
41 C = zeros(N,neps);
42 D = zeros(N,neta);
43
44 % 1. NKPC

```

```

45 A(1,1) = 1;
46 A(1,2) = -kappa;
47 A(1,6) = -betta;
48 A(1,8) = -1;
49
50 % 2. DIS
51 A(2,2) = 1;
52 A(2,3) = sig;
53 A(2,5) = -1;
54 A(2,6) = -sig;
55 A(2,7) = -sig;
56
57 % 3. Foc for inflation
58 A(3,1) = 1;
59 A(3,4) = -1;
60 B(3,4) = -1;
61
62 % 4. Foc for output gap
63 A(4,2) = 1;
64 A(4,4) = kappa/lambdax;
65
66 % 5. AR1 for rn
67 A(5,7) = 1;
68 B(5,7) = rhor;
69 C(5,1) = 1;
70
71 % 6. AR1 for u
72 A(6,8) = 1;
73 B(6,8) = rhou;
74 C(6,2) = 1;
75
76 % 7. Expectation error of inflation
77 A(7,1) = 1;
78 B(7,6) = 1;
79 D(7,2) = 1;
80
81 % 8. Expectation error of inflation
82 A(8,2) = 1;
83 B(8,5) = 1;
84 D(8,1) = 1;
85
86 %
87 % Eigen value decomposition
88 %
89 % Apply generalized schur(QZ) decomposition for S and T.
90 %

```



```

91 % S*E[X(t+1)] = T*X(t)
92 % (Q'*V1*Z')*E[X(t+1)] = (Q'*V2*Z')*X(t)
93 %
94 % where QQ' = ZZ' = I(x NN) .
95 %
96 [S,T,Q,Z] = qz(A,B);
97 s = diag(S);
98 t = diag(T);
99 lambda = abs(t./s) < 1-eps;
100 %
101 % lambda is generalized eigenvalue;
102 % lambda(i) = t(ii)/s(ii)
103 % lambda(i) < 1 is stable root.
104 % Then lambda is a dummy vector which return 1 if eigen value is
    stable.
105 %
106 % reorder by recalling ordqz
107 [S,T,Q,Z] = ordqz(S,T,Q,Z,lambda);
108 s = diag(S);
109 t = diag(T);
110 lambda = abs(t./s) < 1-eps;
111 NS = sum(lambda);
112 %
113 % NS is the number of state variable.
114 %
115
116 % Blanchard and Kahn condition
117 if NS == N-neta
118     disp(' Checking Blanchard and Kahn condition (BKC)... ')
119     disp([' ',num2str(N-NS),' unstable roots'])
120     disp([' ',num2str(neta),' jump variables'])
121     disp([' ',num2str(NS),' stable roots'])
122     disp([' ',num2str(N-neta),' state variables'])
123     disp(' BKC is satisfied.')
124     disp(' ')
125 else
126     disp(' Checking Blanchard and Kahn condition (BKC)...')
127     disp([' ',num2str(N-NS),' unstable roots'])
128     disp([' ',num2str(neta),' jump variables'])
129     disp([' ',num2str(NS),' stable roots'])
130     disp([' ',num2str(N-neta),' state variables'])
131     error('BKC is not satisfied')
132 end
133
134 % Divide stable and unstable block
135 %

```

```

136 % Q'*S*Z'*s(t) = Q'*T*Z'*s(t-1)+Psi*eps(t)+Pi*eta(t)
137 %
138 % where S and T are upper triangular
139 %
140 S11 = S(1:NS,1:NS);
141 S12 = S(1:NS,NS+1:N);
142 S22 = S(NS+1:N,NS+1:N);
143 T11 = T(1:NS,1:NS);
144 T12 = T(1:NS,NS+1:N);
145 T22 = T(NS+1:N,NS+1:N);
146 %
147 % Solution form is
148 %
149 % s(t) = M*s(t-1)+Me*eps(t),
150 %
151 % where
152 %
153 % M = Z*(S11\T11 S11\T12-Phi*T22);0 0)*Z'
154 % Me = Z*(S11\Q1-Phi*Q2);0)*C
155 % Phi = Q1*D*((Q2*D)\eye(size(Q2*D,2)));
156 %
157 Q1 = Q(1:NS,:); % stable block of Q
158 Q2 = Q(NS+1:N,:); % unstable block of Q
159 Phi = Q1*D*((Q2*D)\eye(size(Q2*D,2))); % definition of Phi
160 GG = zeros(N,N);
161 GG11 = S11\T11;
162 GG12 = S11\T12-Phi*T22;
163 GG(1:NS,1:NS) = GG11; % insert GG11 into stable block of GG
164 GG(1:NS,NS+1:N) = GG12; % insert GG12 into unstable block of GG
165 % Mx
166 Mx = Z*GG*(Z\eye(size(Z,2)));
167 GZ = zeros(N,N);
168 GZ(1:NS,:) = S11\Q1-Phi*Q2;
169 % Me
170 Me = Z*GZ*C;
171
172 %
173 % Impulse responses
174 %
175
176 T = 20; % irf period
177 X = zeros(N,T); % box
178
179 varrn = 0; % natural rate shock
180 varu = -2; % cost-push shock
181

```

```

182 eps_t = [varrn,varu];
183
184 % for initial period
185 X(:,1) = Me*eps_t';
186 % following periods
187 for t = 2:T
188     X(:,t) = Mx*X(:,t-1);
189 end
190 %Extradct and set names
191 p = X(1,:);
192 x = X(2,:);
193 i = X(3,:);
194 rn = X(7,:);
195 u = X(8,:);
196
197
198 % Plot impulse responses
199 %
200 % Figure plot
201 %
202 lw = 1; % line width of impulse responses
203 zeroline = zeros(1,T); % steady state lines
204
205 figure(1)
206
207 subplot(3,2,1) % output
208 h = plot(1:T,p); % plot(irf period, irf, irf period, zeroline)
209 set(h(1),'linewidth',lw) % irf line width
210 title('Inflation (\pi)') % irf name
211 xlabel('quarters') % label name of x axis
212 ylabel('percent deviation') % label name of y axis
213
214 subplot(3,2,2) % consumption
215 h = plot(1:T,x);
216 set(h(1),'linewidth',lw)
217 title('Output gap (x)')
218
219 subplot(3,2,3) % investment
220 h = plot(1:T,i);
221 set(h(1),'linewidth',lw)
222 title('Interest rate (i)')
223
224 subplot(3,2,4) % labor
225 h = plot(1:T,rn);
226 set(h(1),'linewidth',lw)
227 title('Natural rate (r^n)')

```

2 OPTIMAL MONETARY POLICY

```
228 |
229 | subplot(3,2,5) % technology
230 | h = plot(1:T,u);
231 | set(h(1),'linewidth',lw)
232 | title('Cosy-push shock (u)')
```

MATLAB Code: Optimal Monetary Policy (Discretion)

```

1 % Optimal monetary policy, Discretion
2 %
3 % NK model
4 %
5 % period loss: 1/2*(pi^2 + x^2)
6 %
7 % structural equations:
8 % x = Ex - sig*(i - Epi -rn)
9 % p = beta*Ep + kpp*x + u
10 % rn = rhor*rn(-1) + epsrn
11 % u = rhou*rn(-1) + epsu
12 %
13 % FOC
14 % p = phip
15 % x = -kappa/lambdax*phip
16
17
18 clear all
19 close
20
21 % Parameters
22 betta = .99; % discount factor
23 sig = 6.25; % intertemporal elasticity
24 kappa = 0.024; % slope of NKPC
25 tht = 7.88; % depreciation rate
26 lambdax = kappa/tht; % weight on output gap
27 rhor = .5; % persistence of natural rate
28 rhou = 0; % persistence of cost shock
29
30 % System of the model:
31 % A*X(t) = B*X(t-1) + C*eps(t) + D*eta(t)
32 % X = [pi, x, i, phip, Ex, Epi, rn, u]
33 % eps(t): Technology shock
34 % eta(t) = [eta_c, eta_y]: Expectation error
35
36 % Prepare matrices
37
38 N = 8; neps = 2; neta = 2; % # of variables, shocks, and expectation
    errors.
39
40 A = zeros(N,N);
41 B = zeros(N,N);
42 C = zeros(N,neps);
43 D = zeros(N,neta);
44

```

```

45 % 1. NKPC
46 A(1,1) = 1;
47 A(1,2) = -kappa;
48 A(1,6) = -betta;
49 A(1,8) = -1;
50
51 % 2. DIS
52 A(2,2) = 1;
53 A(2,3) = sig;
54 A(2,5) = -1;
55 A(2,6) = -sig;
56 A(2,7) = -sig;
57
58 % 3. Foc for inflation
59 A(3,1) = 1;
60 A(3,4) = -1;
61
62 % 4. Foc for output gap
63 A(4,2) = 1;
64 A(4,4) = kappa/lambdax;
65
66 % 5. AR1 for rn
67 A(5,7) = 1;
68 B(5,7) = rhor;
69 C(5,1) = 1;
70
71 % 6. AR1 for u
72 A(6,8) = 1;
73 B(6,8) = rhou;
74 C(6,2) = 1;
75
76 % 7. Expectation error of inflation
77 A(7,1) = 1;
78 B(7,6) = 1;
79 D(7,2) = 1;
80
81 % 8. Expectation error of inflation
82 A(8,2) = 1;
83 B(8,5) = 1;
84 D(8,1) = 1;
85
86 %
87 % Eigen value decomposition
88 %
89 % Apply generalized schur(QZ) decomposition for S and T.
90 %

```

```

91 % S*E[X(t+1)] = T*X(t)
92 % (Q'*V1*Z')*E[X(t+1)] = (Q'*V2*Z')*X(t)
93 %
94 % where QQ' = ZZ' = I(x NN) .
95 %
96 [S,T,Q,Z] = qz(A,B);
97 s = diag(S);
98 t = diag(T);
99 lambda = abs(t./s) < 1-eps;
100 %
101 % lambda is generalized eigenvalue;
102 % lambda(ii) = t(ii)/s(ii)
103 % lambda(ii) < 1 is stable root.
104 % Then lambda is a dummy vector which return 1 if eigen value is
    stable.
105 %
106 % reorder by recalling ordqz
107 [S,T,Q,Z] = ordqz(S,T,Q,Z,lambda);
108 s = diag(S);
109 t = diag(T);
110 lambda = abs(t./s) < 1-eps;
111 NS = sum(lambda);
112 %
113 % NS is the number of state variable.
114 %
115
116 % Blanchard and Kahn condition
117 if NS == N-neta
118     disp(' Checking Blanchard and Kahn condition (BKC)... ')
119     disp([' ',num2str(N-NS),' unstable roots'])
120     disp([' ',num2str(neta),' jump variables'])
121     disp([' ',num2str(NS),' stable roots'])
122     disp([' ',num2str(N-neta),' state variables'])
123     disp(' BKC is satisfied.')
124     disp(' ')
125 else
126     disp(' Checking Blanchard and Kahn condition (BKC)...')
127     disp([' ',num2str(N-NS),' unstable roots'])
128     disp([' ',num2str(neta),' jump variables'])
129     disp([' ',num2str(NS),' stable roots'])
130     disp([' ',num2str(N-neta),' state variables'])
131     error('BKC is not satisfied')
132 end
133
134 % Divide stable and unstable block
135 %

```

```

136 % Q'*S*Z'*s(t) = Q'*T*Z'*s(t-1)+Psi*eps(t)+Pi*eta(t)
137 %
138 % where S and T are upper triangular
139 %
140 S11 = S(1:NS,1:NS);
141 S12 = S(1:NS,NS+1:N);
142 S22 = S(NS+1:N,NS+1:N);
143 T11 = T(1:NS,1:NS);
144 T12 = T(1:NS,NS+1:N);
145 T22 = T(NS+1:N,NS+1:N);
146 %
147 % Solution form is
148 %
149 % s(t) = M*s(t-1)+Me*eps(t),
150 %
151 % where
152 %
153 % M = Z*(S11\T11 S11\T12-Phi*T22);0 0)*Z'
154 % Me = Z*(S11\Q1-Phi*Q2);0)*C
155 % Phi = Q1*D*((Q2*D)\eye(size(Q2*D,2)));
156 %
157 Q1 = Q(1:NS,:); % stable block of Q
158 Q2 = Q(NS+1:N,:); % unstable block of Q
159 Phi = Q1*D*((Q2*D)\eye(size(Q2*D,2))); % definition of Phi
160 GG = zeros(N,N);
161 GG11 = S11\T11;
162 GG12 = S11\T12-Phi*T22;
163 GG(1:NS,1:NS) = GG11; % insert GG11 into stable block of GG
164 GG(1:NS,NS+1:N) = GG12; % insert GG12 into unstable block of GG
165 % Mx
166 Mx = Z*GG*(Z\eye(size(Z,2)));
167 GZ = zeros(N,N);
168 GZ(1:NS,:) = S11\Q1-Phi*Q2;
169 % Me
170 Me = Z*GZ*C;
171
172 %
173 % Impulse responses
174 %
175
176 T = 20; % irf period
177 X = zeros(N,T); % box
178
179 varrn = 0; % natural rate shock
180 varu = -2; % cost-push shock
181

```



```

182 eps_t = [varrn,varu];
183
184 % for initial period
185 X(:,1) = Me*eps_t';
186 % following periods
187 for t = 2:T
188     X(:,t) = Mx*X(:,t-1);
189 end
190 %Extradct and set names
191 p = X(1,:);
192 x = X(2,:);
193 i = X(3,:);
194 rn = X(7,:);
195 u = X(8,:);
196
197
198 % Plot impulse responses
199 %
200 % Figure plot
201 %
202 lw = 1; % line width of impulse responses
203 zeroline = zeros(1,T); % steady state lines
204
205 figure(1)
206
207 subplot(3,2,1) % output
208 h = plot(1:T,p); % plot(irf period, irf, irf period, zeroline)
209 set(h(1),'linewidth',lw) % irf line width
210 title('Inflation (pi)') % irf name
211 xlabel('quarters') % label name of x axis
212 ylabel('percent deviation') % label name of y axis
213
214 subplot(3,2,2) % consumption
215 h = plot(1:T,x);
216 set(h(1),'linewidth',lw)
217 title('Output gap (x)')
218
219 subplot(3,2,3) % investment
220 h = plot(1:T,i);
221 set(h(1),'linewidth',lw)
222 title('Interest rate (i)')
223
224 subplot(3,2,4) % labor
225 h = plot(1:T,rn);
226 set(h(1),'linewidth',lw)
227 title('Natural rate (rn)')

```

2 OPTIMAL MONETARY POLICY

```
228 |
229 | subplot(3,2,5) % technology
230 | h = plot(1:T,u);
231 | set(h(1),'linewidth',lw)
232 | title('Cosy-push shock (u)')
```